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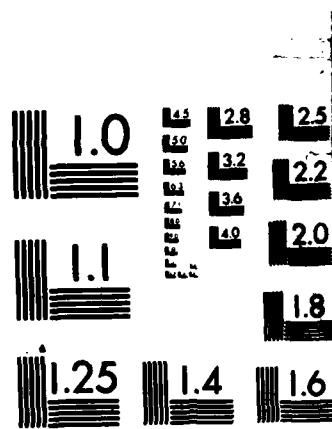
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ALGORITHMS FOR NONLINEAR PROGRAMMING

FINAL REPORT 85-1

BY

DONALD GOLDFARB

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U. S. ARMY RESEARCH OFFICE

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COLUMBIA UNIVERSITY IN THE CITY OF NEW YORK

DEPARTMENT OF INDUSTRIAL ENGINEERING
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <i>See next page.</i> | | |

20.

Several algorithms for solving problems in linear, quadratic, and nonlinear programming, network flows, and facilities location were developed and analyzed.

Our results include:

(1) The analysis of the computational complexity of the problem of determining an optimally sparse representation of the null space of a matrix, and the development of worst-case bounds for the shadow-vertex simplex algorithm and several heuristics for distance constrained discrete facility location problems and conditional covering problems;

(2) The development of efficient algorithms for dense and sparse assignment problems, strictly convex quadratic programming problems, and a nonlinear programming problem that arises when maximizing a correlation coefficient subject to linear constraints; and

(3) The application of iterative methods to large sparse equality-constrained quadratic programs and the development of multiple constraint deletion strategies for active-set algorithms for linearly constrained nonlinear programming problems.

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A. PROBLEM STATEMENT

Our research under Contract number DAAG 29-83-K-0106 focused on the development and analysis of algorithms for mathematical programming, and the production of portable software implementing them. Our work can be categorized according to the areas of linear, quadratic, and nonlinear programming, network flows, and facility location.

In linear programming, several topics were studied including the worst-case and average-case behaviour of the shadow-vertex (parametric objective function) simplex method, the preprocessing of a system of sparse linear constraints to make it sparser, and the determination of an optimally sparse representation of the null space of a matrix.

In quadratic programming we worked on the development of primal algorithms including one that allows several constraints to be dropped simultaneously. Emphasis was on how to implement such algorithms in an efficient and numerically stable manner. We also investigated the application of iterative (SOR-like) methods to quadratic programming problems.

In nonlinear programming, we studied how to extend our work on multiple constraint deletion strategies in quadratic programming to Newton-like active set methods for problems with linear constraints. We also studied methods for solving a problem which arises when maximizing a correlation coefficient subject to linear constraints.

Our research on network flows concentrated on developing and implementing dual simplex algorithms for solving dense and sparse assignment problems.

In the area of facility location, we analyzed and tested several algorithms for both distance constrained discrete facility location problems and conditional covering problems in which facilities are required to cover one another as well as given fixed sites.

B. SUMMARY OF RESEARCH RESULTS

1. Linear Programming:

We have continued our study of the computational complexity of the shadow-vertex simplex algorithm. This method is equivalent to the standard method used for solving a linear program with a parametric objective function. We have shown that this method has exponential worst-case computational bounds. We have also studied the likelihood of such worst-case behaviour using a sign-invariant probabilistic model in collaboration with M. Haimovich.

An algorithm developed by T. McCormick and A. Hoffman that preprocesses a system of sparse linear equations to increase its sparsity has been tested and a refined version appears to have promise of practical applicability. Some results on generalizing these algorithms to problems in which some a priori information is known about the sparsity structure have been obtained.

The determination of an optimally sparse representation of the null space of a matrix has been shown to be NP-complete. Work on developing good heuristic algorithms is in progress.

2. Quadratic Programming:

We developed two active-set primal algorithms for solving strictly convex quadratic programs. Matrix factorizations that were used in the dual algorithm of Goldfarb and Idnani were employed, enabling the algorithms to be both efficient and numerically stable. One of the algorithms used sufficient conditions for simultaneously dropping several constraints from the active set, which it checked with little additional computational effort.

We studied the use of iterative (SOR-like) methods for solving large sparse equality constrained quadratic programs. Necessary and sufficient conditions for the convergence of these methods had previously been obtained by D. Goldfarb. Our attempts at trying to make these methods practicable (e.g., to take full advantage of sparsity) have not been successful.

3. Nonlinear Programming:

We developed several strategies for simultaneously dropping more than one constraint from the active set in active set methods for linearly constrained nonlinear optimization problems. Recurrence formulas were derived which allow one to efficiently determine whether an additional constraint can be dropped without causing the resulting step to violate previously dropped constraints. A "steepest-face" constraint deletion rule for choosing amongst several candidates was derived and its implementation considered.

We showed that the solution to a problem that arises when maximizing a certain correlation coefficient subject to linear constraints can be obtained by solving a sequence of quadratic programs. A FORTRAN code implementing our algorithm has been written and tested.

4. Network Flows:

Two efficient dual simplex algorithms for solving $n \times n$ assignment problems were developed that require at worst $O(n^3)$ operations. Variants of both of these algorithms were devised for solving sparse problems (i.e., ones with inadmissible assignments). Using sophisticated data structures, we showed how to solve problems with m arcs in $O(mn + n^2 \log n)$ time in the worst case. This is as good a bound as is currently known for this problem. Some of these algorithms have been coded in FORTRAN and preliminary tests on randomly generated problems indicate that they are very efficient.

5. Facility Location:

Four greedy heuristics for solving the problem of locating a maximum-weight set of facilities so that no two are closer than a given distance from each other were studied. Worst imaginable worst-case bounds were given for all four

heuristics. Empirical results on a large set of randomly generated test problems indicate, however, that all four perform quite well with none clearly superior.

Seven greedy heuristics were studied for solving the conditional covering problem of finding a minimum set of facility sites that cover not only given demand points but also one another. Extensive computational testing was carried out and it was found that it was better to be greedy on the demand points than on the facility sites. Worst-case error bounds were derived for the two best performing heuristics. These were overly pessimistic when compared with our computational results.

C. PUBLICATIONS

D. Goldfarb, "Efficient Dual Simplex Algorithms for the Assignment Problem," Mathematical Programming (to appear).

D. Goldfarb, "Efficient Primal Algorithms for Strictly Convex Quadratic Programs," in J. P. Hennart, ed., Numerical Analysis, Proceedings, Guanajuato, Mexico, 1984. Lecture Notes in Mathematics (Springer-Verlag, Berlin) to appear.

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D. Goldfarb "Worst-Case Complexity of the Shadow Vertex Simplex Algorithm" IEOR Department Technical Report (1983) Columbia University, New York.

D. Goldfarb and Ph. L. Toint, "Optimal Estimation of Jacobian and Hessian Matrices that Arise in Finite Difference Calculations," Mathematics of Computation, Vol. 43 (1984) pp. 69-88.

S. S. Chaudhry, S. T. McCormick, and I. D. Moon, "Conditional Covering; Greedy Heuristics and Computational Results," submitted to Journal of Computational and Operations Research.

S. S. Chaudhry, S. T. McCormick, and I. D. Moon, "Locating Independent Facilities with Maximum Weight: Greedy Heuristics," submitted to Omega.

D. PARTICIPATING SCIENTIFIC PERSONNEL

Professor Donald Goldfarb (Principal Investigator)

Professor S. Thomas McCormick

Mr. Shih Chih-Tsung (Graduate Research Assistant)

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